Continuity of the Shannon capacity on graphs

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It is known that $\Theta(C_5) = \sqrt{5}$ [Lovász; 1973] but $\Theta(C_7)$ is not known!

Fraction graphs $E_{p/q}$

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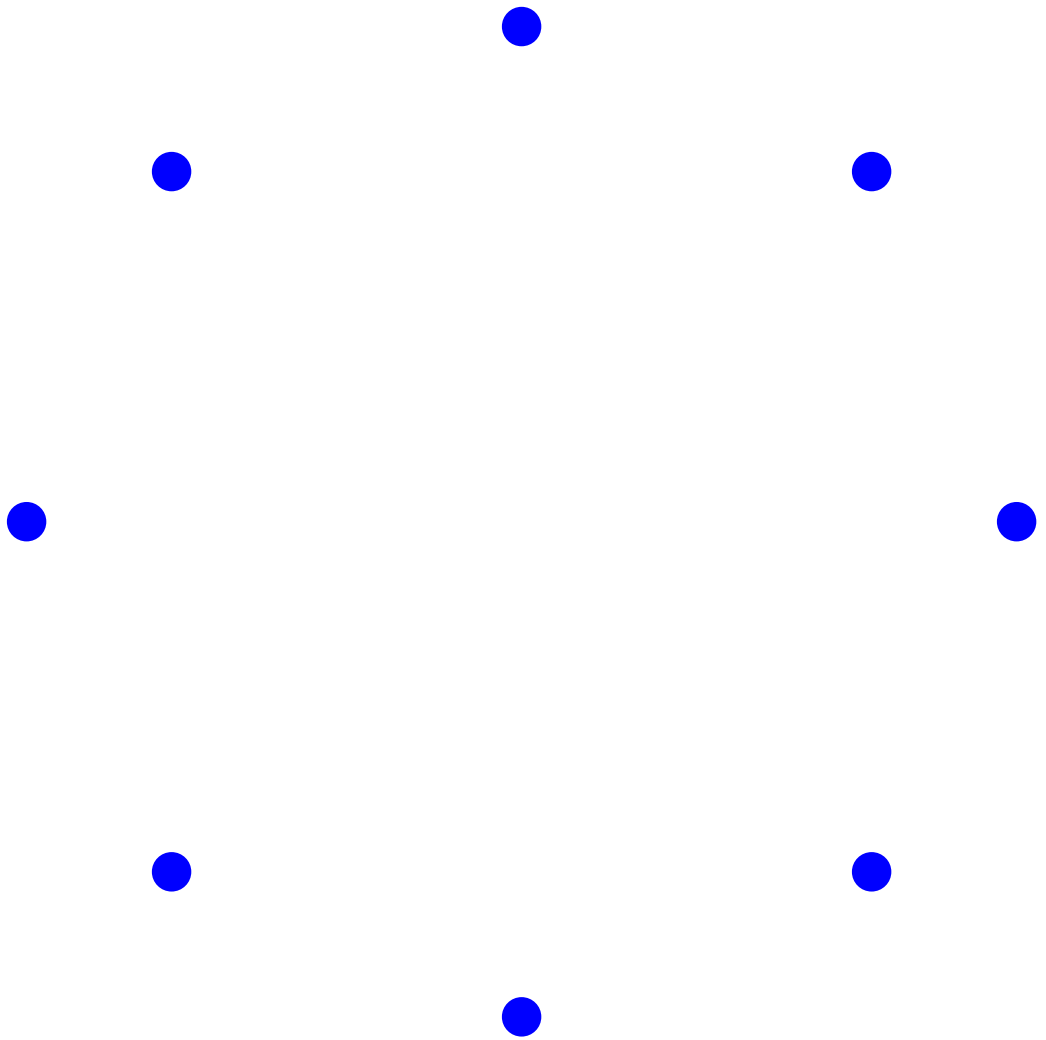

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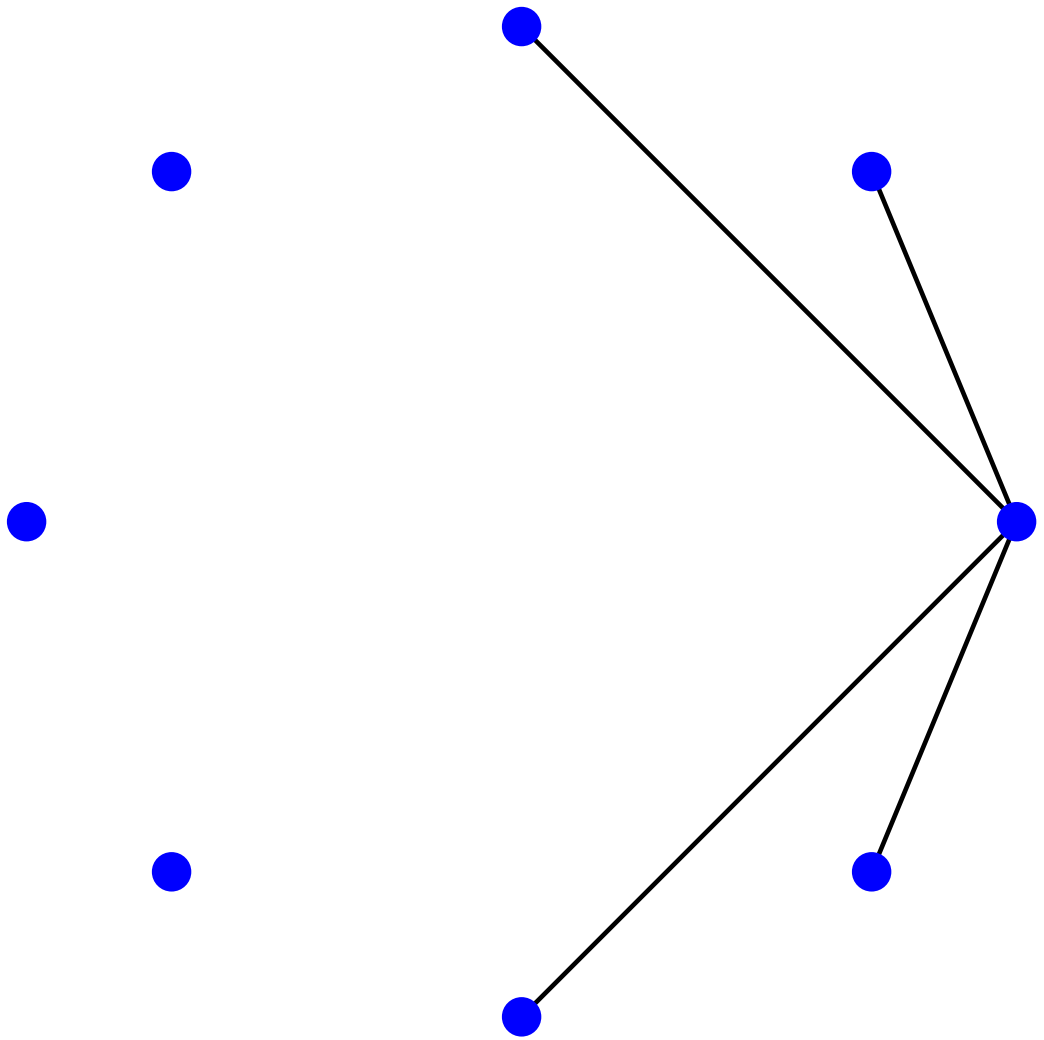

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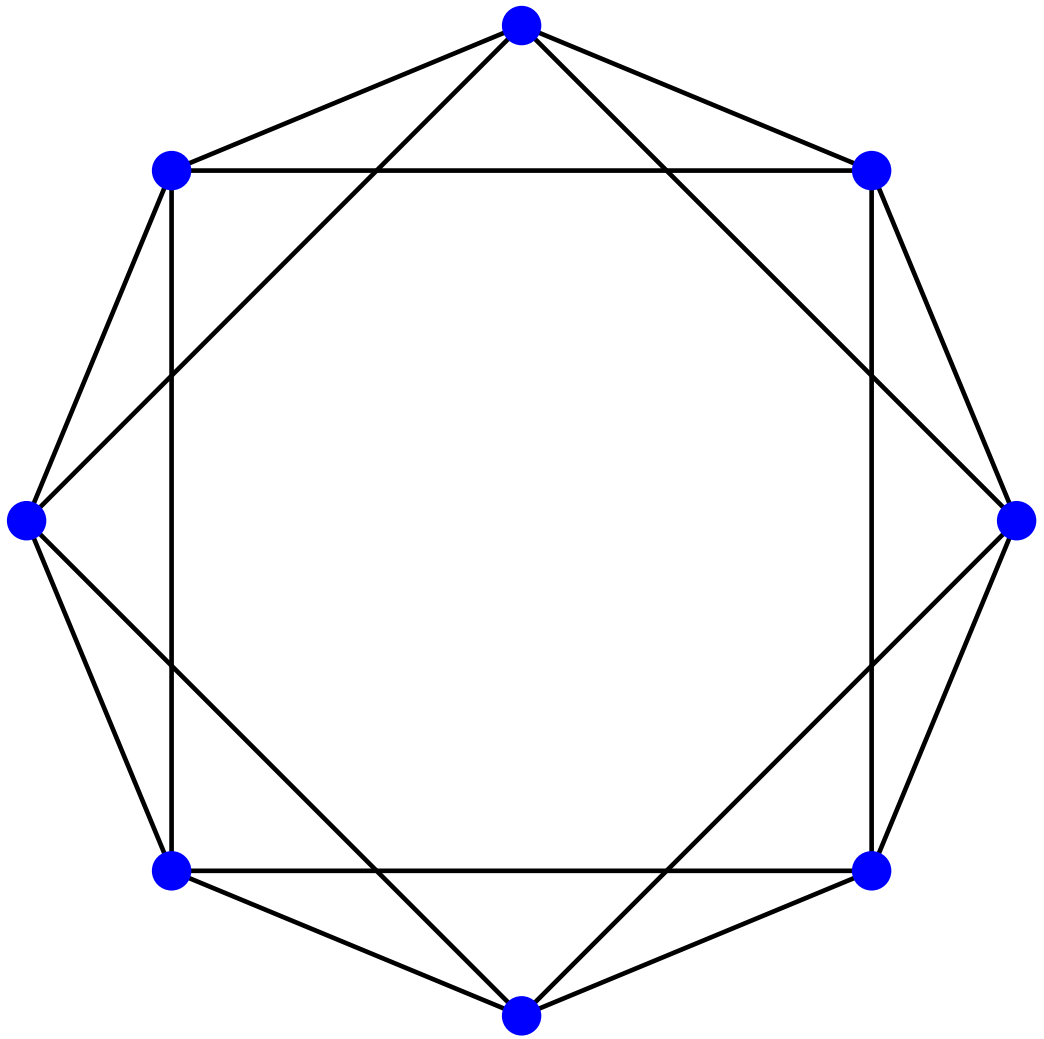
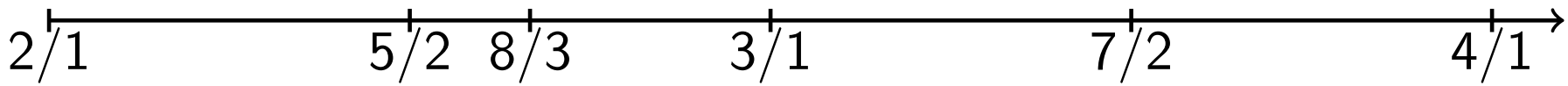
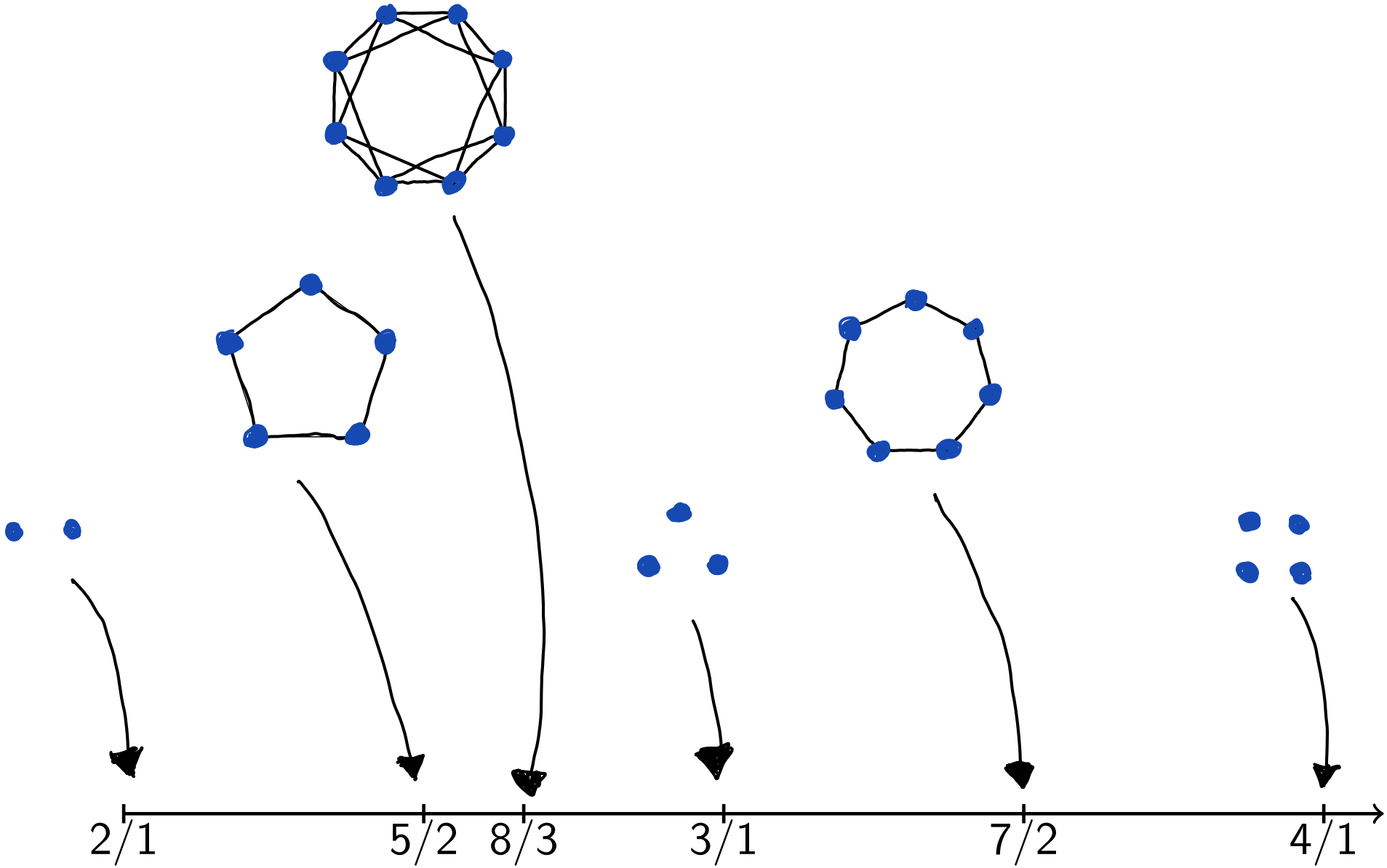



2/1
5/2
8/3
3/1
7/2
4/1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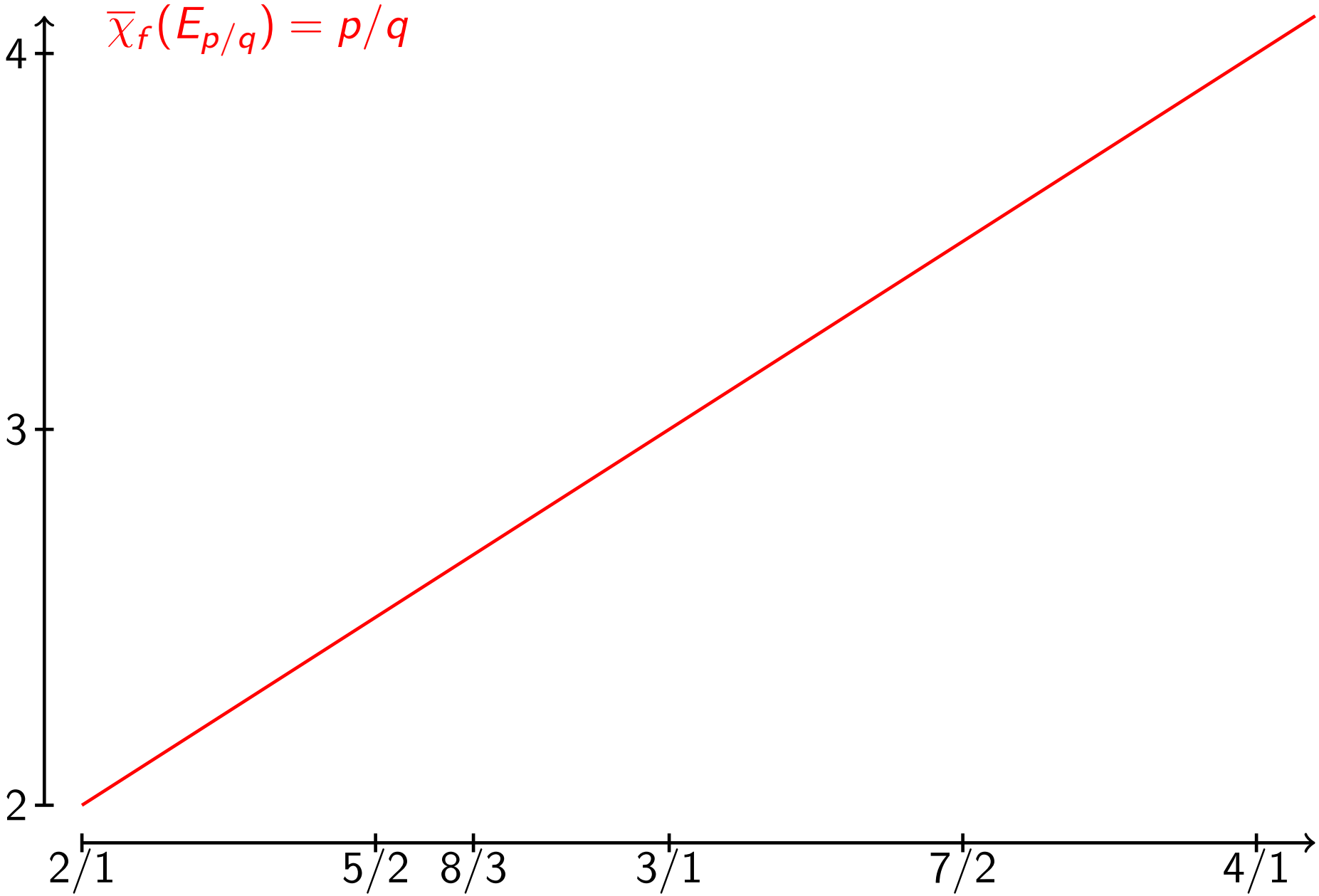
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4
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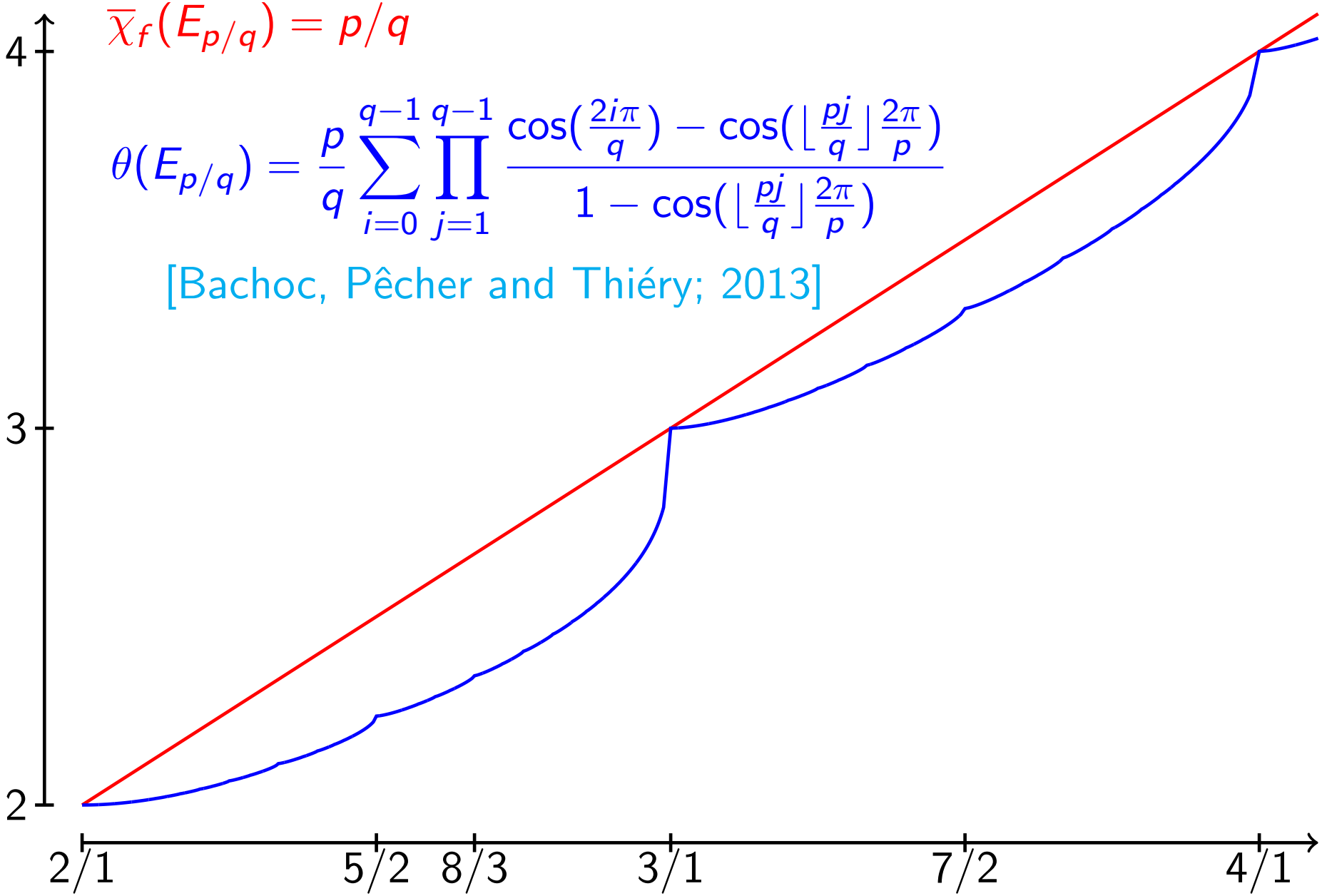
[Bachoc, Pêcher and Thiéry; 2013]



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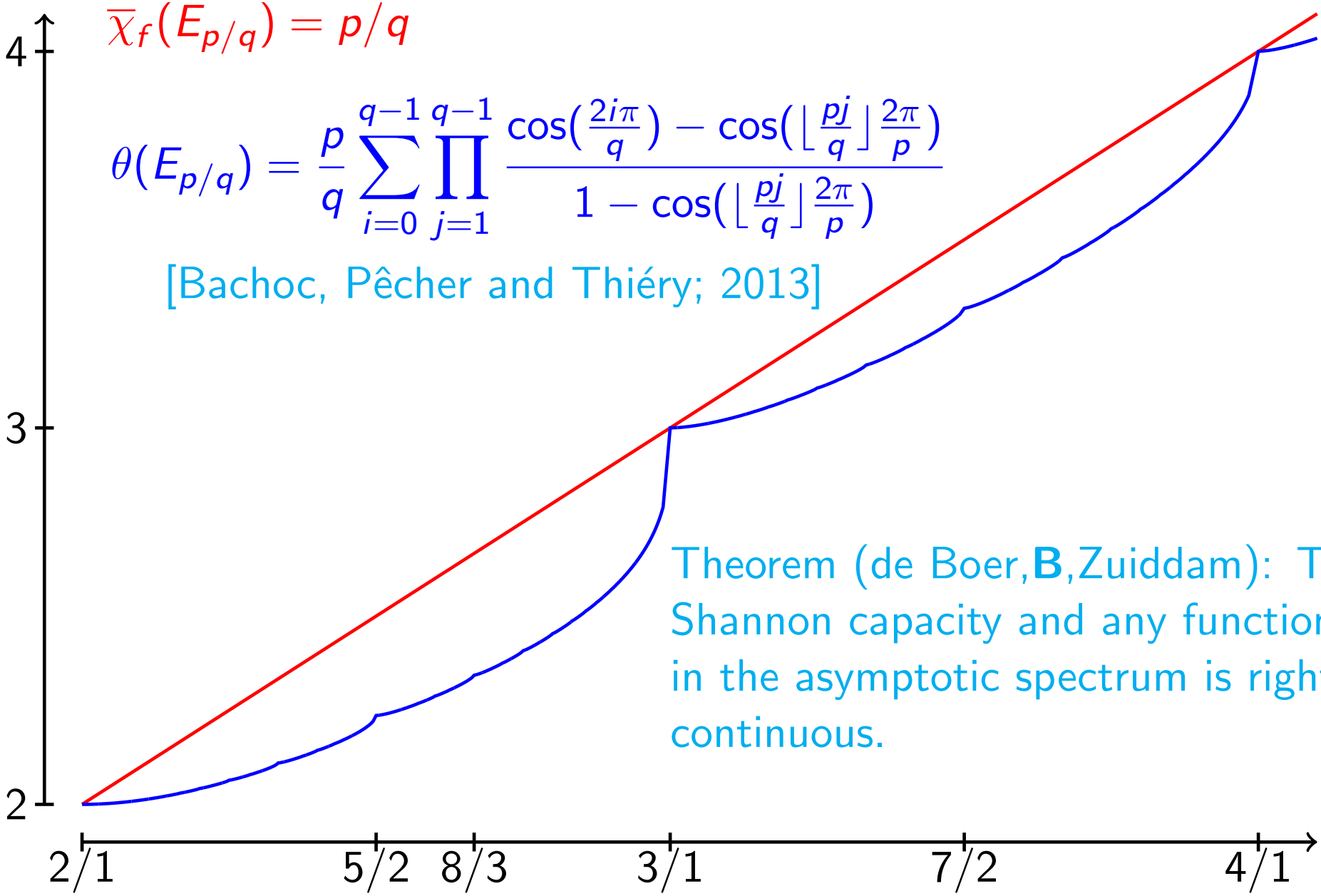

Theorem (de Boer,**B**,Zuiddam): The Shannon capacity and any function in the asymptotic spectrum is right-continuous.

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